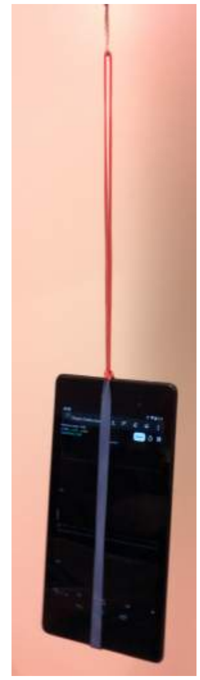
Simple Harmonic Motion: Spring coefficient and damping.

Linear spring constant

The period, T, of oscillation of a mass, m, hanging from a spring with a linear spring constant, k, is $T = 2\pi\sqrt{\frac{m}{k}}$. Measuring mass and period allows the spring constant to be calculated.



Procedure:

1. Measure the mass of your phone or tablet. [The Nexus 7 has a mass of 0.290 kg.]

2. Attach the phone to the bottom of a spring and hang the spring from the top. You may need to add more mass so the spring stretches sufficiently to oscillate smoothly. [Note: For small oscillations the restoring force of a rubber band is approximately linear and so can be used as a spring. However, if the rubber band is stretched too far the restoring force is no longer linear. Rubber bands also experience hysteresis if stretched slowly; the force measured for the same length is slightly different as the band stretches than when the band relaxes.]

3. Start the Mobile Science Harmonic Motion app* and tap *Collect*.

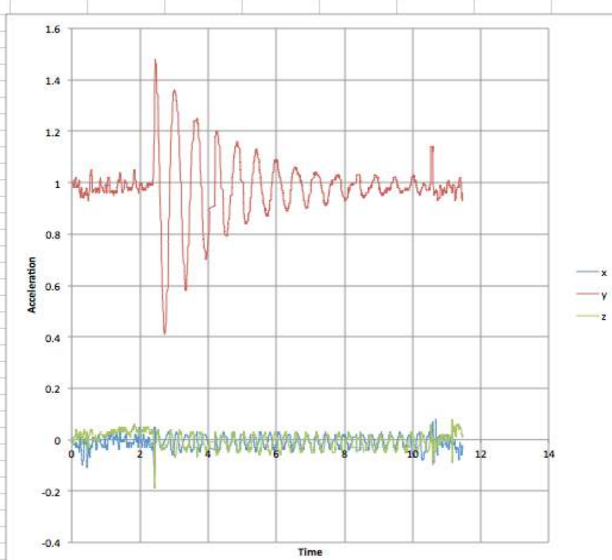
4. In the Physics Toolbox app, tap the *Record* button, pull the device down and release. After several bounces, *Stop* the data collection.

5. Use the back button to return to the Harmonic Motion app** and tap *Analyze*. The app loads the data into a spreadsheet *copy.xls* file using *input.xls* as a template. Your graph should be similar to the figure below.

6. The peaks are the acceleration at the bottom of each oscillation; the valleys (minimums) are the acceleration at the top of each oscillation. Two times the average peak to peak time is the period, T. Find the average period for your spring using the peak times in column G of spreadsheet.

7. The spring constant is given by $k = m\left(\frac{2\pi}{T}\right)^2$ where m is the total mass of phone plus any additional weight. What is the spring constant of your spring?

time	x	y	z
0.009	0.01	1.01	-0.02
0.01	0.01	1.01	-0.01
0.025	0	1.01	-0.02
0.026	-0.01	1.01	-0.01
0.027	-0.01	1	-0.01
0.028	-0.01	1	0
0.056	-0.02	0.99	-0.01
0.058	-0.02	0.99	-0.01
0.059	-0.02	0.98	-0.01
0.06	-0.02	0.98	0
0.061	-0.02	0.98	0
0.063	-0.02	0.98	0.01
0.064	-0.02	0.98	0.01
0.082	-0.02	0.99	0.01
0.083	-0.02	1	0.01
0.084	-0.02	1.01	0.01
0.085	-0.02	1.01	0.01
0.123	-0.01	1.02	0.02
0.124	-0.02	1.02	0.02
0.125	-0.02	1.02	0.02
0.127	-0.02	1.01	0.01
0.128	-0.02	1.01	0.02
0.129	-0.03	0.99	0.03
0.13	-0.03	0.98	0.04
0.131	-0.02	0.98	0.04
0.133	-0.02	0.98	0.04
0.134	-0.01	0.98	0.03
0.145	-0.02	0.98	0.02
0.147	-0.02	0.98	0.01
0.176	-0.02	0.99	0
0.177	-0.02	0.98	0.01
0.179	-0.03	0.99	0.01
0.18	-0.03	0.99	0.01
0.181	-0.03	0.99	0.02
0.182	-0.03	0.99	0.02



Damping coefficient b

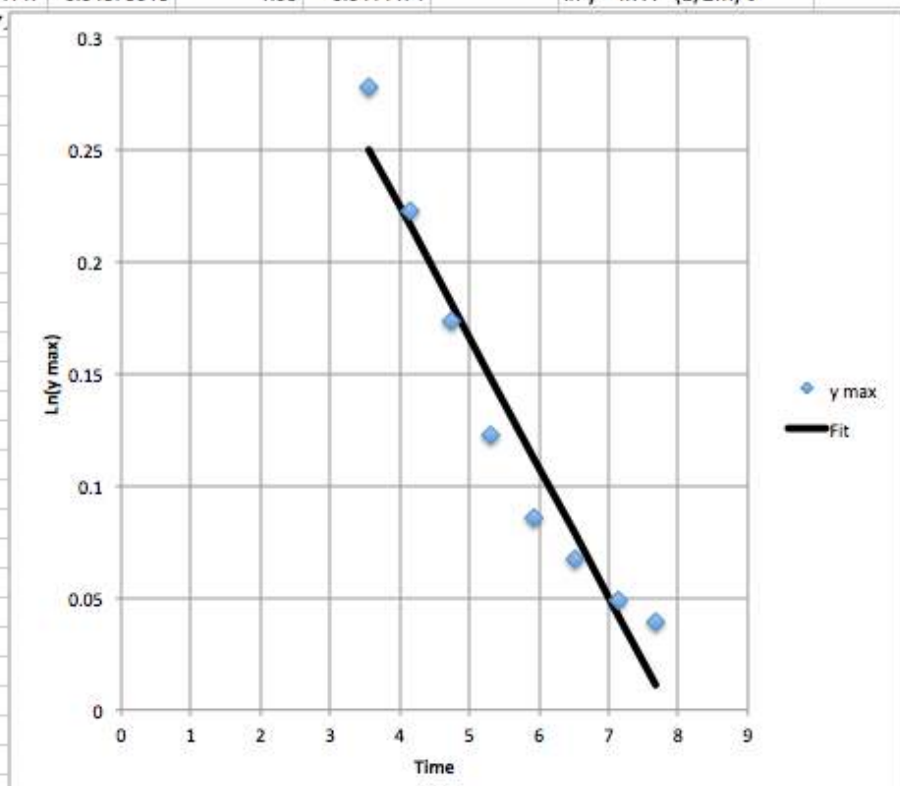
Due to damping, the maximum amplitude of the oscillation will decay exponentially; $y = Ae^{-\frac{b}{2m}t}$ where b is the damping coefficient and m is the hanging mass. This equation can be linearized by taking a natural logarithm to give $lny = lnA - \frac{b}{2m}t$. A plot of lny versus t is a straight line with slope $-\frac{b}{2m}$.

1. The app performs the linearization and plots the result in the spreadsheet. The graph should be similar to the one shown below.

2. Using the constant given by the equation, calculate the damping coefficient, b. [In the graph below, $\frac{b}{2m} = 0.058$ so $b = 2m * 0.058$. The initial amplitude, A can be found from $lnA = 0.457$].

3. The spreadsheet data (the file *copy.xls* has the current data) can be downloaded to a computer by email. The template, *input.xls*, spreadsheet can downloaded, modified, and uploaded to the device to change the format of the graph or add a different analysis to the template.

G	H	I	J	K	L	M	N	O
time	ln(y max)	y max	Fit	A = exp(0.457)		y = exp(-0.058 * x + 0.457)		
3.562	0.27763174	1.32	0.25011967	b/2m = -0.058		R^2 = 0.940		
4.143	0.22314355	1.25	0.21630133					
4.743	0.17395331	1.19	0.18137705					
5.312	0.12221763	1.13	0.14825719					
5.93	0.0861777	1.09	0.11228518		y=A exp(-bt/2m)			
6.514	0.06765865	1.07	0.07829221					
7.147	0.04879016	1.05	0.0414471		ln y = ln A - (b/2m) t			
7.								



* Harmonic Motion app: https://play.google.com/store/apps/details?id=coedu.ius.harmonicmotion
**Physics Toolbox Accelerometer app:
https://play.google.com/store/apps/details?id=com.chrystianvieyra.android.physicstoolboxaccelerometer